Home

Right-angled triangles having a perimeter p

The objective of this article is to explain how to calculate the right-angled triangles having a given perimeter.

(x, y, z) is a right-angled triangle in (x, y).
We seek the whole numbers x, y and z such as $x + y + z = p$ (p whole known).
Nous cherchons les entiers x, y et z tel que $x + y + z = p$ (p entier connu).

My formula 4a applied to the right-angled triangles is :

$$x^2 + \left(\frac{x^2 - d^2}{2d}\right)^2 = \left(\frac{x^2 + d^2}{2d}\right)^2$$

(d is a factor of x^2. $0<d<x$)

$x + y + z = p.$

$x + \frac{x^2-d^2}{2d} + \frac{x^2-d^2}{2d} = p.$

$x + \frac{x^2}{d} = p.$

→ $x^2 + dx - dp = 0.$

It is an equation of 2nd degree, therefore $x = \frac{-d+\sqrt{d^2+4dp}}{2}$

So that x is whole, it is necessary that $\sqrt{d^2 + 4dp}$ is whole. To demonstrate it, we must use my formula 2a (see detail in this page for understand the rest).

To demonstrate that $d^2 + 4dp$ is square perfect, we calculate $\mathbf{\Delta_2 = 16p^2}$, and we seek factors d_2 of $16p^2$ ($0<d_2<4p$).
We obtain :

$$d = \frac{-4p + \frac{16p^2 + d_2^2}{2d_2}}{2}$$

$$x = \frac{-d + \sqrt{d^2 + 4dp}}{2}$$

$$y = \frac{x^2 - d^2}{2d}$$

$$z = y + d$$

d_2 : factor of $16p^2$; ($0<d_2<4p$)
d_2 and $\frac{16p^2}{d_2}$ must be even
$\frac{d_2}{2}$ and $\frac{8p^2}{d_2}$ must have the same parity
d and x must have the same parity

Now, we have all the tools to calculate x, y and z by knowing only p. Let us see that in detail with an example.

We seek the right-angled triangles, if they exist, having a perimeter p = 180.
Let us proceed by step, with the explanations of the table of calculation :

1. We calculate $\Delta_2 = \mathbf{16p^2}$ → $\Delta_2 = 518400$;
2. We seek all factors d_2 of 518400, d_2 must be even and $\mathbf{0<d_2<4p}$;
3. We check the conditions of validity of each factor d_2 :
   In the column Δ_2/d_2, 2025 is red because it is odd. Thus the factor 256 will not be retained.
   In the columns $d_2/2$ and $\Delta_2/2d_2$, certain values are red because they do not have the same parity. Thus the factors d_2 correspondents will not be retained.
4. Then, we calculate d for each value selected of d_2 : $\mathbf{d} = \dfrac{-4p + \frac{16p^2 + d_2^2}{2d_2}}{2}$;
5. And $\mathbf{x} = \dfrac{-d + \sqrt{d^2 + 4dp}}{2}$;
   Certain values of the columns d and x are red because they do not have the same parity. Thus useless to calculate y and z corresponding because they are not integers ;
6. Then, we calculate $\mathbf{y} = \dfrac{x^2 - d^2}{2d}$ and $\mathbf{z = y + d}$

	Δ_2		Conditions			d	x	y	z
p	$16p^2$	d_2	Δ_2 / d_2	$d_2 / 2$	$\Delta_2 / 2d_2$	$((16p^2+d_2^2)/2d_2-4p)/2$	$(-d+\mathrm{sqrt}(d^2+4dp))/2$	$(x^2 - d^2) / 2d$	y + d
180	518400	2	259200	1	129600				
		4	129600	2	129600	32041	179	-16020	16021
		6	86400	3	43200				
		8	64800	4	32400	15842	178	-7920	7922
		10	51840	5	25920				
		12	43200	6	21600	10443	177	-5220	5223
		16	32400	8	16200	7744	176	-3870	3874
		18	28800	9	14400				
		20	25920	10	12960	6125	175	-3060	3065
		24	21600	12	10800	5046	174	-2520	2526
		30	17280	15	8640				
		32	16200	16	8100	3698	172	-1845	1853
		36	14400	18	7200	3249	171	-1620	1629
		40	12960	20	6480	2890	170	-1440	1450
		48	10800	24	5400	2352	168	-1170	1182
		50	10368	25	5184				
		54	9600	27	4800				
		60	8640	30	4320	1815	165	-900	915
		64	8100	32	4050	1681	164		
		72	7200	36	3600	1458	162	-720	738
		80	6480	40	3240	1280	160	-630	650
		90	5760	45	2880				
		96	5400	48	2700	1014	156	-495	519
		100	5184	50	2592	961	155	-468	493
		108	4800	54	2400	867	153	-420	447
		120	4320	60	2160	750	150	-360	390
		128	4050	64	2025				
		144	3600	72	1800	576	144	-270	306
		150	3456	75	1728				
		160	3240	80	1620	490	140	-225	265
		162	3200	81	1600				
		180	2880	90	1440	405	135	-180	225
		192	2700	96	1350	363	132		

200	2592	100	1296	**338**	**130**	-144	194
216	2400	108	1200	**294**	**126**	-120	174
240	2160	120	1080	**240**	**120**	-90	150
256	**2025**						
270	1920	**135**	**960**				
288	1800	144	900	**162**	**108**	-45	117
300	1728	150	864	**147**	**105**	-36	111
320	1620	160	810	**125**	**100**		
324	1600	162	800	**121**	**99**	-20	101
360	1440	180	720	**90**	**90**	0	90
384	1350	**192**	**675**				
400	1296	200	648	**64**	**80**	18	82
432	1200	216	600	**48**	**72**	30	78
450	1152	**225**	**576**				
480	1080	240	540	**30**	**60**	45	75
540	960	270	480	**15**	**45**	60	75
576	900	288	450	**9**	**36**		
600	864	300	432	**6**	**30**	72	78
640	810	**320**	**405**				
648	800	324	400	**2**	**18**	80	82
720							

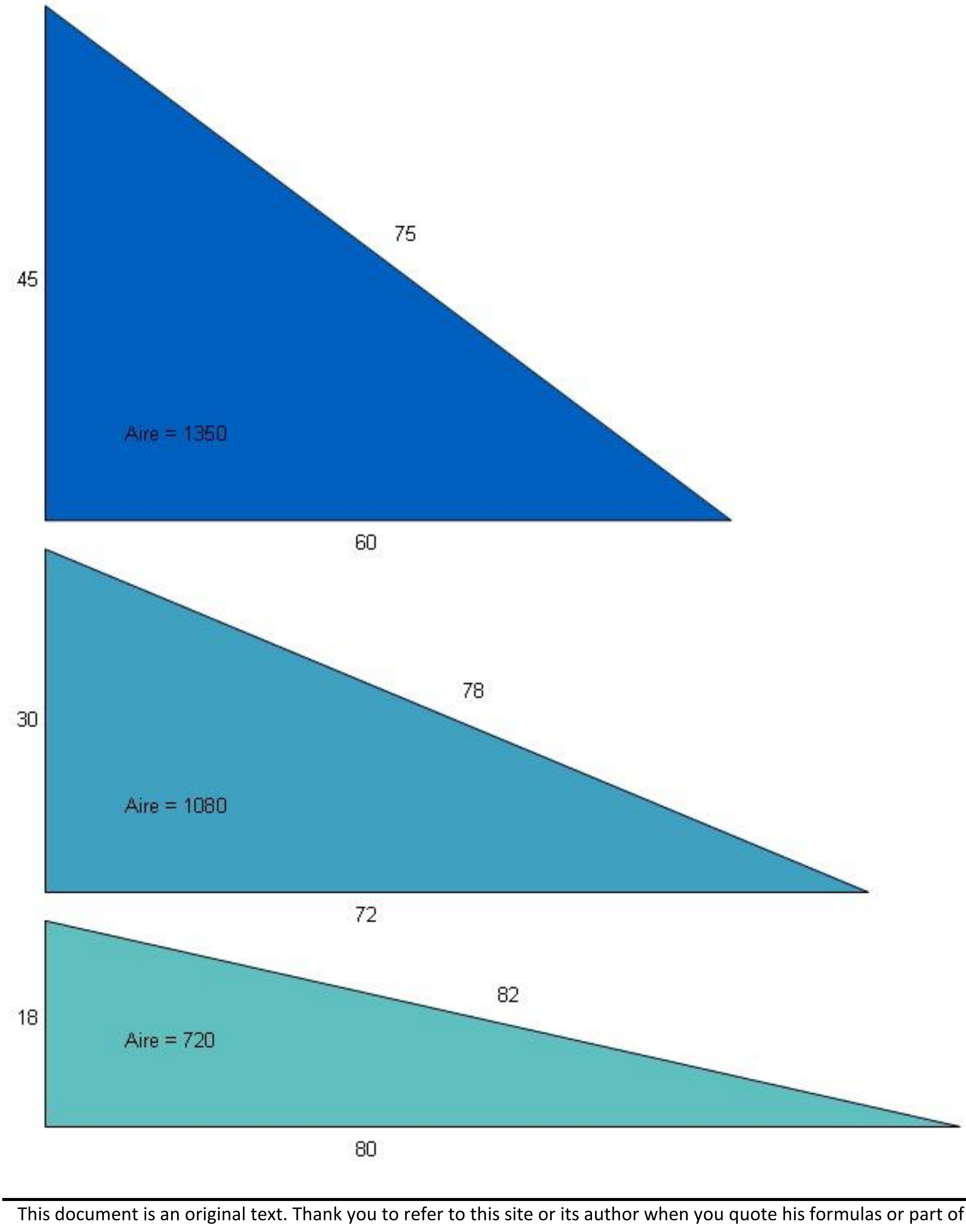
We obtain 33 results with $x^2 + y^2 = z^2$ and $x + y + z = 180$:

$179^2 + (-16020)^2 = 16021^2$ and $179 - 16020 + 16021 = 180$
...
$18^2 + 80^2 = 82^2$ and $18 + 80 + 82 = 180$.

On these 33 results, only 6 triples have the positive values for x, y and z. These triples are those obtained by $\mathbf{2p < D_2 < 4p}$, they are in green in the table.
These 6 triples are duplicated because the formula distinguishes between (18, 80, 82) and (80, 18, 82) for example.
We thus have 3 different triples whose 3 values are positive.
So there are 3 right-angled triangles whose perimeter is equal to 180 :
(18, 80, 82), (30, 72, 78) and (45, 60, 75).

Note :

Before launching you in tests, will know that the perimeter of a right-angled triangle is always an even number.
$p = x + \frac{x^2}{d}$.
If x is even, x^2/d, by definition, must be even. Thus p is even
If x is odd, d and x^2/d are necessarily odd. Thus p is even.


75
45
Aire = 1350
60
78
30
Aire = 1080
72
82
18
Aire = 720
80